WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

PHSACOR01T-PHYSICS (CC1)

Time Allotted: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No.1 and any *two* questions from the rest

1. Answer any ***ten*** questions from the following: $2\times10=20$

(a) Prove that the series $x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots$ is convergent for $0<x<1$.

(b) Find the general solution of the equation

$$2x\frac{dy}{dx}+y=2xe^{5/2}$$

(c) A function $f(x)$ is defined by

$$\begin{aligned} f(x) &= \cos x \text{ for } x\geq 0 \\ &= -\cos x \text{ for } x<0 \end{aligned}$$

Is $f(x)$ continuous at $x=0$? Give reasons.

(d) If $f(x)=|x|$, show that $f(0)$ is a minimum although $f'(0)$ does not exist.

(e) Prove $\nabla^2 f(r)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$

(f) Calculate the Laplacian of the scalar field $\ln(x^2+y^2)$.

(g) Expand $f(x)=\frac{1}{x-2}$ in a Taylor series about the point $x=1$.

(h) Wronskian of two functions is $\omega(t)=t\sin^2 t$. Are the functions linearly independent or linearly dependent? Explain.

(i) If $\vec{A}=2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{B}=\hat{i}-2\hat{j}+\hat{k}$, find a vector of magnitude 5 perpendicular to both $\vec{A}$ and $\vec{B}$.

(j) Show that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x, y, z)=c$ where c is a constant.

(k) Find an expression for ds^2 in curvilinear co-ordinates u, v, w. Then determine ds^2 for the special case of an orthogonal system.

(l) An integer is chosen at random from the first 200 positive integers. What is the probability that the integer chosen is divisible by 6 or 8?

(m) A distribution function is given by $f(x)=\dfrac{1}{\pi\sqrt{A^2-x^2}}$ where x is the random variable. Find the value of $\langle |x| \rangle$. (Assume 'A' is a constant).

(n) Show that, for large number of trials, Binomial distribution yields Poisson distribution.

2. (a) Check if the following differential equation $\dfrac{dy}{dx}-y\tan x=e^x\sec x$ is exact. Hence, solve the equation. 1+3

(b) Show that $\vec{A}\cdot(\vec{B}\times\vec{C})$ is an absolute value equal to the volume of a parallelepiped with sides $\vec{A},\vec{B}$ and $\vec{C}$. Hence, find the condition for these vectors to be coplanar. 2+1

(c) Find the extrema of $f(x, y)=5x-3y$ subject to the constraint $x^2+y^2=136$. 3

3. (a) Solve: $\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+2y=e^{2x}\sin x$. 6

(b) Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is a solenoidal only if $n=-3$. 4

4. (a) Suppose that X is exponentially distributed with $\lambda=3$. $2\frac{1}{2}+2\frac{1}{2}+1$

(i) What is $P\{X>2\}$?

(ii) What is $P\{X>5\,|\,X>3\}$?

(iii) What did you notice about these two answers? Is it a coincidence?

(b) A person makes steps of length 'l' is just likely to step forwards as backwards. Prove that after n steps in this random walk, the person will have gone forward a distance rl with a probability $\left(\frac{1}{2}\right)^n\, {}^nC_{\frac{n+r}{2}}$. 4

5. (a) Find the expression of Laplacian in cylindrical coordinate system. 4

(b) Prove that $\oint u\nabla v\cdot d\lambda=-\oint v\nabla u\cdot d\lambda$ 3

(c) Evaluate $\oint_C y^3dx-x^3dy$ where C is the positively oriented circle of radius 2 centered at the origin. 3

———×———