लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2019

MTMACOR01T-MATHEMATICS (CC1)

Time Allotted: 2 Hours

Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5=10$

(a) Evaluate $\lim\limits_{x\to\infty}\dfrac{x^4}{e^x}$ using L'Hospital's rule.

(b) Write the equation $xy=1$ in terms of a rotated rectangular $x'y'$-system if the angle of rotation from the x-axis to the x'-axis is 45°.

(c) Find the differential equation satisfied by the family of curves given by $c^2+2cy-x^2+1=0$, c being the parameter of the family.

(d) Find the length of a quadrant of the circle $r=2a\sin\theta$.

(e) Find the curves passing through (0, 1) and satisfying $\sin\left(\dfrac{dy}{dx}\right)=c$.

(f) Find the values of b and g such that the equation $4x^2+8xy+by^2+2gx+4y+1=0$ represents a conic without any centre.

(g) Test whether the equation $x\,dx+y\,dy+\dfrac{x\,dy-y\,dx}{x^2+y^2}=0$ is exact or not.

(h) Find the singular solutions of

$$9\left(\frac{dy}{dx}\right)^2(2-y)^2=4(3-y).$$

2. (a) Find the evolute of the parabola $y^2=8x$. 3

(b) If $x=\tan(\log y)$, prove that $(1+x^2)y_{n+1}+(2nx-1)y_n+n(n-1)y_{n-1}=0$. 5

3. (a) Find the asymptotes of the curve $x^3-2y^3+xy(2x-y)+y(x-y)+1=0$. Prove that these asymptotes cut the curve in three points which lie on a line. 3+1+1

(b) Find the envelope of the family of straight lines, which together with the line segments intercepted by the coordinate axes form triangles of equal area. 3

4. (a) Show that $\int_0^1 x^m (\log x)^n \, dx = (-1)^n \frac{n!}{(m+1)^{n+1}}$, where $m \geq 0$ and n is a positive integer. 4

(b) Find the surface area of the reel formed by the revolution of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ about the tangent at the vertex. 4

5. (a) Find the values of c for which the plane $x + y + z = c$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$. 3

(b) Show that the section of the surface $yz + zx + xy = a^2$ by the plane $lx + my + nz = p$ will be a parabola if $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$. 3

(c) Prove that if a straight line meets a conicoid in three points, then it will be a generator of the conicoid. 2

6. (a) Reduce $11x^2 + 4xy + 14y^2 - 26x - 32y + 23 = 0$ to its normal form using orthogonal transformations. 4

(b) A variable plane passes through a fixed point. Show that the locus of the foot of the perpendicular from the origin to the plane is a sphere. 4

7. (a) Determine the arc length of the parametric curve given by the following set of parametric equations. You may assume that the curve traces out exactly once for the given range of t's : $x = 3t + 1$, $y = 4 - t^2$, $-2 \leq t \leq 0$. 5

(b) For the conic described by the polar equation $r = \frac{12}{4 + 5\cos\theta}$ with focus at the origin, find the directrix, eccentricity and nature. 1+1+1

8. (a) Solve the differential equation: 4

$$x(y\,dx + x\,dy)\cos\left(\frac{y}{x}\right) = y(x\,dy - y\,dx)\sin\left(\frac{y}{x}\right)$$

(b) Show that the equation of the curve whose slope at any point (x, y) is equal to $y + 2x$ and which passes through the origin is $y = 2(e^x - x - 1)$. 4

9. (a) Solve: $(xy^2 + 2x^2y^3)\,dx + (x^2y - x^2y^2)\,dy = 0$. 4

(b) Solve: $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$. 4

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